End-bearing capacity of driven piles in sand using the stress characteristics method: analysis and implementation

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Abstract: The method of stress characteristics has been employed to compute the end-bearing capacity of driven piles. The dependency of the soil internal friction angle on the stress level has been incorporated to achieve more realistic predictions for the end-bearing capacity of piles. The validity of the assumption of the superposition principle while using the bearing capacity equation based on soil plasticity concepts, when applied to deep foundations, has been examined. Fourteen pile case histories were compiled with cone penetration tests (CPT) performed in the vicinity of different pile locations. The end-bearing capacity of the piles was computed using different methods, namely, static analysis, effective stress approach, direct CPT, and the proposed approach. The comparison between predictions made by different methods and measured records shows that the stress-level-based method of stress characteristics compares better with experimental data. Finally, the end-bearing capacity of driven piles in sand was expressed in terms of a general expression with the addition of a new factor that accounts for different factors contributing to the bearing capacity. The influence of the soil nonassociative flow rule has also been included to achieve more realistic results.

Key words: bearing capacity, cone penetration tests, failure, friction angles, piles, plasticity.

Résumé : La méthode des caractéristiques des contraintes a été utilisée pour calculer la capacité portante en pointe de pieux enfoncés. La dépendance de l'angle de friction interne du sol sur le niveau de contrainte a été incluse pour obtenir des prédictions plus réalistes de la capacité portante en pointe des pieux. La validité de l'hypothèse du principe de superposition lorsqu'on utilise l'équation de capacité portante basée sur les concepts de plasticité des sols a été examinée pour l'application au cas des fondations profondes. Les études de cas de 14 pieux ont été compilées, dans lesquelles des essais de pénétration du cône (EPC) ont été réalisés dans l'environnement immédiat des différents pieux. La capacité portante en pointe des pieux a été calculée à l'aide de différentes méthodes, soit l'analyse statique, l'approche des contraintes effectives, l'EPC direct et l'approche proposée. La comparaison entre les prédictions obtenues avec les différentes méthodes et les données mesurées démontrent que la méthode des caractéristiques des contraintes, basée sur le niveau des contraintes, se compare le mieux avec les données expérimentales. Finalement, la capacité portante en pointe de pieux enfoncés dans le sable est exprimée comme une expression générale avec l'ajout d'un nouveau facteur qui considère les différents facteurs qui contribuent à la capacité portante. L'influence de la loi d'écoulement non associative du sol a aussi été incluse afin d'obtenir des résultats plus réalistes.

Mots-clés : capacité portante, essais de pénétration du cône, rupture, angles de friction, pieux, plasticité.

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Introduction

The estimation of the bearing capacity of deep foundations has always been a major issue in geotechnical engineering. There are several methods for determining the bearing capacity of deep foundations, namely, theoretical formula and (or) static analysis (Vesić 1963; Janbu 1976; Kulhawy 1984; Poulos 1989), an application of in situ test records (Meyerhof 1976; Schmertmann 1978; Eslami and Fellenius 1995, 1997), dynamic methods (Goble and Rausche 1979; Rausche et al. 1985; Fellenius 2006), and interpretation of full-scale pile load tests (Fellenius 1990). While design strategies based on in situ tests and dynamic methods are sometimes costly and time consuming, the theoretical approach, namely static analysis, is often chosen as a first step for performing design. The theoretical approaches have been developed based on a variety of assumptions and simplifications. Like in the case of shallow foundations, the well-known triple-*N* formula of Terzaghi (1943) has been the essential basis for determining the bearing capacity for the base and (or) tip of deep foundations; the expression has the following general form:

$$q_{\mathrm{ult}} = cN_{\mathrm{c}} + qN_{\mathrm{q}} + 0.5\gamma BN_{\gamma} \quad [1]$$

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where q_{ult} is the ultimate bearing capacity, c is cohesion, q is surcharge pressure, B is the foundation width, γ is the soil unit weight, and N_c, N_q, and N_γ coefficients are the bearing capacity factors that are functions of soil friction angle. These factors are very sensitive to the variation of friction angle. Unlike the first two factors, that is, N_c and N_q, the third factor (N_γ) is the most disputable one. There are several values for the third factor suggested by different authors (Meyerhof 1963; Bolton and Lau 1993; Kumar and Khatri 2008; Kumar 2009). In deep foundations, however, the contribution of the third term is seldom significant and can be neglected because of relatively small pile diameters (Bowles 1996). Therefore, the basic ultimate base capacity is regarded as $cN_c + qN_q$. Except for preconsolidated clays and cemented sands, this equation will take the form qN_q. As the variation of soil friction angle has a major influence on N_q, it becomes important to examine the dependency of soil friction angle on the stress level. The dependency of soil friction angle on the stress level has been well observed and reported (Meyerhof 1950; Bolton 1986), and its influence on the bearing capacity of shallow foundations has been widely studied (Bolton and Lau 1989; Clark 1998; Jahanandish et al. 2010). In deep foundations, this effect seems to be of greater importance.

Besides the dependency of soil friction angle on the stress level, the effect of the flow rule becomes equally significant as the nonassociative flow rule generally decreases the bearing capacity (Frydman and Burd 1997; Michalowski 1997). Michalowski (1997) used equivalent terms for soil friction angle and cohesion, based on the recommendation of Drescher and Detournay (1993), and then calculated the bearing capacity factors for nonassociated soils. The upper bound limit analysis was utilized, and the results indicated a significant decrease in the ultimate bearing capacity factors for soil friction angles greater than 30°. For smaller friction angles, this effect becomes less significant, but still remains equally important. There are rather few attempts to consider the effect of the nonassociative flow rule on the ultimate bearing capacity of the pile base.

In this study, the method of stress characteristics has been employed to predict the ultimate bearing capacity of the pile base. The effect of stress level has been duly incorporated in the analysis. The results have been verified with experimental data obtained from pile load test records. The results are further compared with various other methods, namely, static analysis, effective stress approach, and direct cone penetration test (CPT) method. The effect of the nonassociative flow rule has also been investigated.

Stress level effect on soil shear strength

It is well understood that the soil shear strength is stress-dependent and the Mohr–Coulomb yield surface, if chosen as the yield criterion, is not linear (Meyerhof 1950; Lee and Seed 1967; Holtz and Kovacs 1981; Bolton 1986; Clark 1998; Kumar et al. 2007). Bolton (1986) proposed the following relationship indicating the dependency of the peak soil friction angle on the stress level:

$$[2a] \qquad \phi_{\max} = \phi_{\text{c.s.}} + 0.8\nu$$

$$[2b] \qquad \phi_{\max} = \phi_{\text{c.s.}} + 3I_R \qquad \text{(for triaxial strain)}$$

$$[2c] \qquad I_R = D_r[Q - \ln(\sigma)] - R$$

where $\phi_{\max}$ is the peak friction angle, $\phi_{\text{c.s.}}$ is the critical state friction angle, ν is the dilation angle, I_R is the dilatancy index, D_r is the soil relative density (in decimal form), σ is the effective stress (in kPa), and Q and R are constants. Bolton (1986) recommended $Q = 10$ and $R = 1$, which differ slightly from the values suggested later by Kumar et al. (2007).

Clark (1998) proposed a simpler equation

$$[3] \qquad \phi = A(\sigma')^M$$

In this equation, ϕ is the peak friction angle as a function of σ'; A is a factor that can be considered as the peak friction angle measured at unit normal or confining pressure, σ'; σ' is the effective confining pressure (in a triaxial test) or normal stress (in a direct shear test); and M is an exponent. This equation requires a set of standard direct shear or triaxial shear tests to determine the parameters.

The dependency of ϕ on σ' will be incorporated in the next section to investigate its influence on the ultimate bearing capacity of foundations and its importance in the development of the bearing capacity formula.

Bearing capacity and nonlinearity of the Mohr–Coulomb failure envelope

There are two major arguments on the validity of Terzaghi's (1943) general bearing capacity equation because it assumes the superposition of three different terms obtained individually and then combined in the final expression. Further, the effect of stress level and nonlinearity of Mohr–Coulomb yield criterion needs to be included in the derivation of the contributing factors; in particular, for relatively large and (or) deep foundations as well as highly complex stress patterns in soil continua.

Nonlinearity of the Mohr–Coulomb yield criterion can result from different reasons, for instance, stress level dependency of the soil friction angle (Clark 1998). On account of this, the bearing capacity of foundations would not linearly increase with foundation size, as suggested by the third term of the bearing capacity equation. The increase in the bearing capacity shows a nonlinear tendency due to the variation of the third factor itself. The bearing capacity, N_γ, decreases with an increase in the width and (or) diameter of the foundation. This aspect has been thoroughly investigated in the available literature for shallow foundations (Bolton and Lau 1989, 1993; Clark 1998; Cerato 2005; Cerato and Lutenegger 2007; Yamamoto et al. 2009; Jahanandish et al. 2010; Veiskarami et al. 2011).

On the other hand, Davis and Booker (1971) made stringent checks on the validity of the superposition assumption and showed that it leads to a safe design. Bolton and Lau (1993) also showed that this assumption is conservative for materials obeying a linear Mohr–Coulomb yield envelope. In contrast, there is no guarantee of the validity of such an assumption for nonlinear Mohr–Coulomb yield criterion.

For circular foundations, Cox (1962) did not apply the superposition assumption and instead, introduced a dimensionless factor as follows:

$$G = \frac{0.5\gamma B}{\sigma_0 \tan\phi} \quad [4]$$

where σ_0 is the atmospheric pressure. The assumption made by Cox (1962) was mainly intended to show that the solution is dependent in a nonlinear fashion on the self-weight and the surcharge pressure. Bolton and Lau (1993) proposed to treat σ_0 as the stress applied to the plane surface around the foundation instead of the atmospheric pressure and introduced their new dimensionless parameter. They suggested using the following dimensionless factor and the combined bearing capacity factor, $N_{q\gamma}$, displaying the importance of both surcharge and self-weight effects:

$$\Omega = \frac{q}{0.5\gamma B} \quad [5a]$$

$$N_{q\gamma} = \frac{q_{ult}}{0.5\gamma B + q} \quad [5b]$$

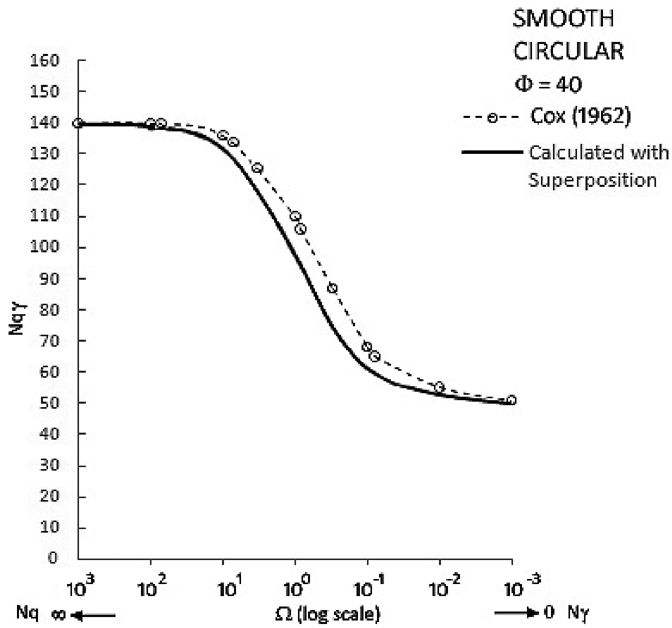
In limits, $N_{q\gamma}$ approaches either N_q or N_γ when Ω approaches infinity or zero, respectively. It is shown in Fig. 1 based on the analyses made by Cox (1962) and Bolton and Lau (1993).

In summary, the superposition assumption can be dropped, at least for cases in which both the surcharge and self-weight effects are important. In shallow foundations, the contribution of the surcharge pressure is negligible, in particular for relatively large foundations and, hence, the role of the third bearing capacity factor, N_γ, is much more important (Jahanandish et al. 2010). In contrast, for deep foundations, the surcharge and the self-weight effects are of similar importance. Also, the Mohr–Coulomb yield surface would not remain linear because of very intensive stress levels and complex stress patterns around the pile toe. Therefore, for determining the bearing capacity of the pile base it would be more rational that the bearing capacity not simply be decomposed into different components.

Method of stress characteristics

The method of stress characteristics is a renowned method for solving plasticity problems in soil mechanics in which both equilibrium and yield equations are satisfied simultaneously. Derivation of the equations for the method of stress characteristics can be found in many sources (Sokolovskii 1960; see Harr 1966 or Sabzevari and Ghahramani 1972 for constant soil shear strength parameters; see Anvar and Ghahramani 1997 for variable shear strength parameters). Only the final forms of these equations are presented here. The basic concept of this method is to transform the equilibrium-yield equations onto a curvilinear coordinate system defined by two directions, namely, positive and negative stress characteristics. Using the standard notation of Anvar and Ghahramani (1997), these two directions can be defined by the following equations:

Fig. 1. Variation of the combined bearing capacity factor, $N_{q\gamma}$, with Ω (from Bolton and Lau 1993).



$$\frac{dz}{dx} = \tan(\theta \pm \mu) \quad [6]$$

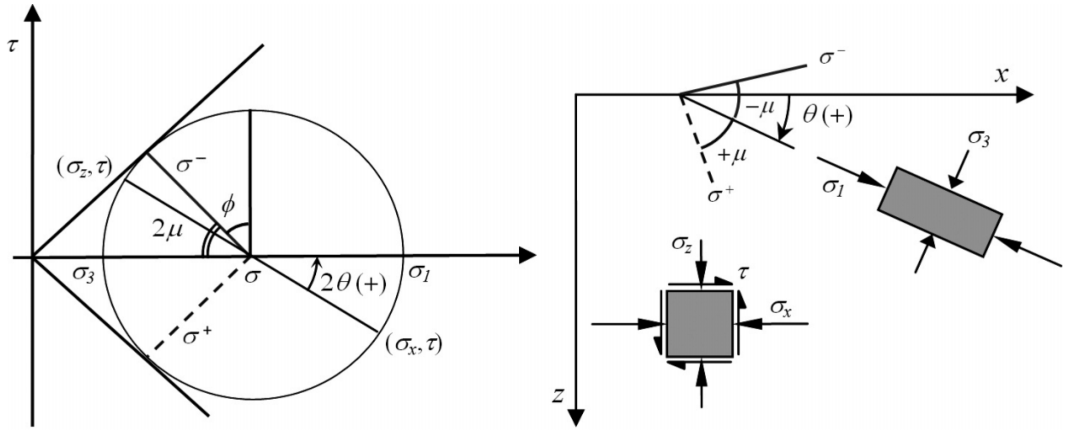
$$\mu = \frac{\pi}{4} - \frac{\phi}{2} \quad [7]$$

In these equations, x and z are measures of the horizontal and vertical distances, respectively; ϕ is the soil friction angle; and θ is the angle between the direction of the major principal stress and the positive x-axis (according to Fig. 2). By these definitions, the stress characteristic equations along these two directions for a field of variable soil friction angle and cohesion intercept would be as follows (Anvar and Ghahramani 1997):

$$[8] \quad \begin{cases} \text{Positive direction (along } \sigma^+\text{):} \\ d\sigma + 2(\sigma\tan\phi + c)\,d\theta = -X(\tan\phi\,dz - dx) + Z(\tan\phi\,dx + dz) + (\sigma - c\tan\phi)\left(\frac{\partial\phi}{\partial z}dx - \frac{\partial\phi}{\partial x}dz\right) + \left(\frac{\partial c}{\partial z}dx - \frac{\partial c}{\partial x}dz\right) \\ \text{Negative direction (along } \sigma^-\text{):} \\ d\sigma - 2(\sigma\tan\phi + c)\,d\theta = +X(\tan\phi\,dz + dx) - Z(\tan\phi\,dx - dz) - (\sigma - c\tan\phi)\left(\frac{\partial\phi}{\partial z}dx - \frac{\partial\phi}{\partial x}dz\right) - \left(\frac{\partial c}{\partial z}dx - \frac{\partial c}{\partial x}dz\right) \end{cases}$$

where X and Z are body forces in horizontal and vertical directions, respectively, c is the soil cohesion intercept, and σ is the mean normal stress at a point: $\sigma = (\sigma_1+\sigma_3)/2$. The stress characteristics directions are shown in Fig. 2.

Fig. 2. Directions of the stress characteristics on Mohr's circle of stress and the major and minor principal stresses.



Therefore, eqs. [6] and [8] result in a system of partial differential equations consisting of four equations in four unknowns, namely, x, z, σ, and θ, which should be solved simultaneously. The finite difference method can be reasonably used to solve these equations. Solution techniques can be found in the literature (Harr 1966; Anvar and Ghahramani 1997).

As stated earlier, the effect of flow rule is very significant in an attempt to estimate the limit load based on plasticity equations. The requirements of the lower-bound limit theorem necessitate the normality to hold and, therefore, an associated flow rule is the basis of the stress characteristics method. For coaxial flow rule materials, obeying a nonassociated flow rule, based on the shearing response of a cohesionless elastic – perfectly plastic material, the apparent friction angle can be equivalently defined as:

$$[9] \qquad \tan\phi^* = \frac{\cos\nu \sin\phi}{1 - \sin\nu \sin\phi}$$

where ϕ^* is the apparent friction angle (equivalent friction angle used in a nonassociated flow rule analysis) and ν is the angle of dilation (Vermeer 1990; Drescher and Detournay 1993; Michalowski 1997). In a cohesionless medium, for given values of ϕ and ν, this equation can be applied to nonassociative materials by making use of ϕ^* in the method of stress characteristics.

New approach for pile end capacity prediction

It is evident that when a pile is driven into the soil the surrounding soil is compressed, and as a consequence, a lateral stress will be imposed on the pile shaft. There are several possible failure mechanisms identifiable at the pile tip. They are shown typically in Fig. 3 (Vesić 1967).

There are several methods used to determine the pile bearing capacity. Besides the different approaches for effective and total stress analyses (ESA and TSA, respectively), when the long-term bearing capacity is required, that is, based on the effective stress parameters, these methods include the static analysis and direct or indirect use of the in situ tests as described in the literature (Bowles 1996; CGS 2006; Eslami and Gholami 2006; Fellenius 2006). A summary of methods for the end-bearing capacity of piles in sand, implemented in this study are presented in Table 1.

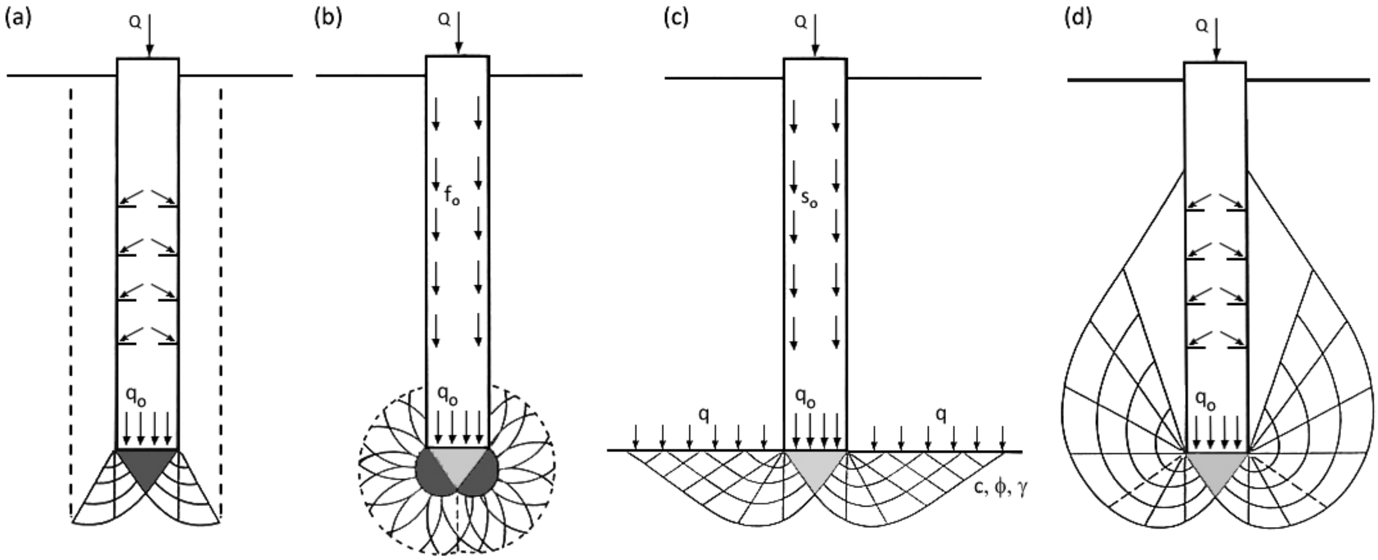
The end-bearing capacity of piles is aimed to be computed, based on the proposed theoretical approach, using the following assumptions:

- The pile is in the ultimate state, that is, vertical displacement required for full mobilization of the ultimate load has been reached.
- The skin resistance has been previously and fully mobilized.
- Pile end bearing capacity is independent of the shaft resistance (they are determined independently).
- Once the limit load is reached and plastic zones are formed, the stress state on the pile shaft (vertical boundary) can be found by one the following assumptions:
    - Soil is in the passive state, that is, some horizontal strains may occur and the soil adjacent to the pile would be compressed.
    - Soil is in the k_0 state, that is, no horizontal displacements occur at the pile interface (a relatively rigid pile).
    - Soil is free of horizontal stresses (this is the case in cast-in-place piles).
- Soil is assumed to be isotropic.
- Soil obeys an associated flow rule; only the yield surface may change its size proportional to the stress level the normality (associated flow rule) holds and the angle of dilation is equal to the soil internal friction angle.

End-bearing capacity of the pile was computed based on the mechanisms and boundary conditions shown in Fig. 4. It was noted that the state of the soil mass adjacent to the pile shaft shows better agreement with experimental data when a k_0 state is assumed.

It is also remarkable that the stress distribution pattern around the pile toe is very complex and highly variable from point to point. If the soil is assumed to be at yield, it would be more realistic to take the effect of stress level on the maximum mobilized soil friction angle into account. It can be done by making use of the equations relating the stress level to the maximum soil friction angle. For example, Bolton's (1986) equation can be applied, relating the maximum friction angle of sand based on the critical state friction angle and the soil relative density, D_r. Both of these parameters can be easily obtained by direct use of the CPT results. For the peak friction angle, Robertson and Campanella (1983) suggested the following equation based on the CPT data:

Fig. 3. Different failure patterns around the pile tip assumed by different researchers: (*a*) Berezantzev and Yaroshenko (1962), Vesić (1963); (*b*) Bishop et al. (1945), Skempton et al. (1953); (*c*) Prandtl (1920), Reissner (1924), Caquot (1934), Bulsman (1935), Terzaghi (1943); (*d*) De Beer (1945), Jáky (1948), Meyerhof (1951).



$$[10] \qquad \tan\phi_{\mathrm{p}} = \frac{1}{2.68}\left[\log\left(\frac{q_{\mathrm{c}}}{\sigma'_{\mathrm{vc}}}\right) + 0.29\right]$$

This equation is suggested for the "unaged moderately compressible predominantly quartz sand." In this equation, q_c is the cone tip resistance and σ'_{vc} is the effective vertical stress.

Also, the relative density (in %) can be obtained from the CPT data using the equation suggested by Bolton and Gui (1993):

$$[11] \qquad \mathrm{Dr} = 0.2831\left(\frac{q_{\mathrm{c}} - \sigma_{\mathrm{v}}}{\sigma'_{\mathrm{v}}}\right) + 32.964$$

where q_c is the cone tip resistance, and σ_v and σ'_v are the total and effective vertical stresses, respectively.

In recent decades, "the cavity expansion theory" has been employed for a wide range of problems dealing with piles and CPT soundings (Mayne 1991; Salgado, et al. 1997; Russel and Khalili 2006). For the end-bearing capacity of piles, the analogy of cavity expansion theory can also be applied with a closed-form solution presented by Carter et al. (1986) or Yu and Houlsby (1991) with similar results. In their approaches, like shallow foundations, an assumption is made that there is a rigid cone of soil formed beneath the pile base inclined at an angle α, which is a function of the soil friction angle (Vesić 1975). Beyond this rigid conical region, the soil is under an isotropic pressure equal to the limit pressure for a spherical cavity expansion. Numerical analyses made by Collins et al. (1992) showed that appropriate values of the soil friction and dilation angles are the averaged values between the initial and ultimate values, that is, those corresponding to the peak and constant volume friction angles. Peak and constant volume friction and dilation angles can be related to each other by Bolton's (1986) equation as a function of the stress level. This has been discussed by Randolph et al. (1994) to achieve appropriate parameters for the cavity expansion analogy. Therefore, evidence shows that the bearing capacity factor, N_t, can be related to the stress level. This latter dependency of N_t on the stress level or some related measures of the stress level (like the embedment depth) will be discussed further in the next sections.

In summary, the toe-bearing capacity of piles in sand can be defined in either of the following two general equations, depending on the method used (Randolph et al. 1994; CGS 2006; Fellenius 2006):

$$[12a] \qquad r_{\mathrm{t}} = N_{\mathrm{t}} q_{\mathrm{z}} \quad \text{Static analysis approaches}$$

$$[12b] \qquad r_{\mathrm{t}} = k_{\mathrm{c}} q_{\mathrm{c}} \quad \text{Direct CPT method}$$

In these equations, r_t is the unit toe resistance, N_t is some bearing capacity factor depending on the soil type and geometry, q_z is the surcharge pressure at the level of the pile base embedment, k_c is a factor relating the end bearing capacity of the pile to the CPT cone resistance, and q_c is the cone tip resistance.

The proposed approach involves the solution of the plasticity equations using the method of stress characteristics for a driven pile problem. This problem consists of a vertical boundary condition (i.e., the pile shaft) and a horizontal boundary condition (i.e., the pile toe) at which the magnitude of the ultimate stress is required. Variation of the soil friction angle with the stress level has been considered in the equations. A computer code in MATLAB was developed to solve the stress characteristic equations by a finite difference method. Therefore, in the proposed theoretical approach the stress-level-based method of stress characteristics has been employed to solve the equations numerically, subject to an appropriate boundary condition.

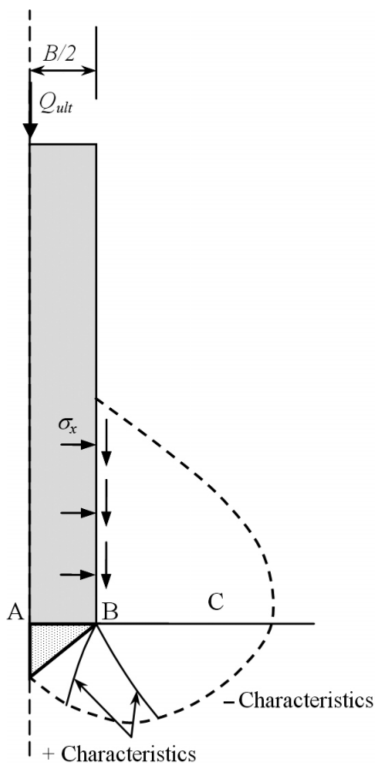
Comparison of the proposed approach with static analysis based methods

In this section, the end-bearing capacity has been computed for some different cases assuming a variable soil friction angle as a function of the relative density and the

Table 1. Summary of different methods in design of the end bearing capacity of piles in sand.

Classification	Method	End-bearing capacity	Remarks
Static analysis	Vesić (1975)	$r_t = \eta q\left(N'_q - 1\right)d_q$ $\eta = (1 + 2k_0)/3$	q, effective vertical overburden pressure at pile toe; N'_q, bearing capacity factor; k_0, coefficient of lateral soil pressure at rest; d_q, depth factor defined as $d_q = 1 + 2\tan\phi'\left(1 - \sin\phi'\right)^2 \tan^{-1}(L/B)$ where ϕ' is the soil friction angle, L is the pile embedment depth in the dense sand strata, and B is the pile diameter.
Static analysis	Janbu (1976)	$r_t = q\left(N'_q - 1\right)d_q$	(Same as Vesić 1975)
Direct CPT	Meyerhof (1976, 1983)	$r_t = C_1C_2q_{ca}$ $C_1 = [(B + 0.5)/2B]^n$ $C_2 = D/10B$	C_1, scale effect modification (C_1 = 1 if B < 0.5 m); C_2, penetration into dense strata modification (C_2 = 1 if D > 10B)C_1, scale effect modification (C_1 = 1 if B < 0.5 m); q_{ca}, arithmetic average of the cone resistance over a zone extending from a depth of 1B beneath the pile toe up to a height of 4B above the pile toe; B, pile diameter; n, an exponent corresponding to the soil packing, 1 for loose (q_c < 5 MPa), 2 for medium (5 MPa < q_c < 12 MPa), and 3 for dense (q_c > 12 MPa) sand; D, pile embedment depth in dense strata.
Direct CPT (CPTu)	Unicone (1997) (Eslami and Fellenius 1997)	$r_t = C_tq_{Eg}$	C_t, toe correlation coefficient; q_{Eg}, geometric average of the cone resistance over the influence zone (extending from a depth of 4B beneath the pile toe up to a height of 8B above the pile toe for heterogeneous soils and 4B below and above for homogeneous soils) after correction for pore pressure on shoulder and adjustment for the effective stress.
ESA	CGS (2006)	$r_t = N_t\sigma'_t$	N_t, bearing capacity factor; σ'_t, vertical effective stress at the pile toe.

Fig. 4. Boundary conditions of Bolton and Lau (1993) with a straight rigid cone.



critical state friction angle. These values are compared with those obtained from the analytical methods suggested by Vesić (1975) and Janbu (1976).

For comparison purposes, the values of the soil critical state friction angle, $\phi_{c.s.}$, and the relative density of sand, D_r, were taken to be 30° and 50%, respectively; only the geometrical parameters were changed. Figure 5*a* shows the stress characteristics net around the pile tip for a 10 m long and 0.5 m diameter pile. Figure 5*b* shows the variation of the maximum mobilized soil friction angle in the same case. It is evident that the zone of the highest stress adjacent to the pile toe exhibits the lowermost value of the soil friction angle, marginally close to the critical state value. In contrast, higher soil friction angle is mobilized around the pile shaft with the lowermost stress level. Figure 6 shows the results of the analyzed cases for 5 m long and 30 m long piles. According to this figure, it can be observed that, for longer piles with higher stress levels at the toe, the mobilized friction angles are lower in comparison to those of shorter piles.

The results were then compared to those obtained using different conventional methods based on the static analysis, which are commonly used in evaluation of the end-bearing capacity of driven piles. These methods comprise the following:

- Vesić (1975), based on the peak friction angle;
- Vesić (1975), based on the critical state friction angle;
- Janbu (1976), based on the peak friction angle;
- Janbu (1976), based on the critical state friction angle;
- CGS (2006), lower bound based on the minimum recommended N_t;
- CGS (2006), upper bound based on the maximum recommended N_t.

Figure 7 shows the results of the analyzed cases for piles of variable lengths in a medium dense sand (D_r = 50%) and the critical state friction angle equal to 30°. Soil was assumed to be in the passive state on the pile shaft. The results show that Janbu's (1976) method provides higher estimates than Vesić's (1975) method when the peak friction angle was assumed. Moreover, there is a significant difference between the results when the critical state friction angle is assumed instead of the peak friction angle. The results obtained using the proposed approach, considering the stress level effect on soil friction angle, lie within the wide band between the two families of curves for

Fig. 5. Pile toe from a close view: (*a*) stress characteristics lines (L = 10 m, D = 0.5 m) and (*b*) variation of the maximum mobilized friction angles.

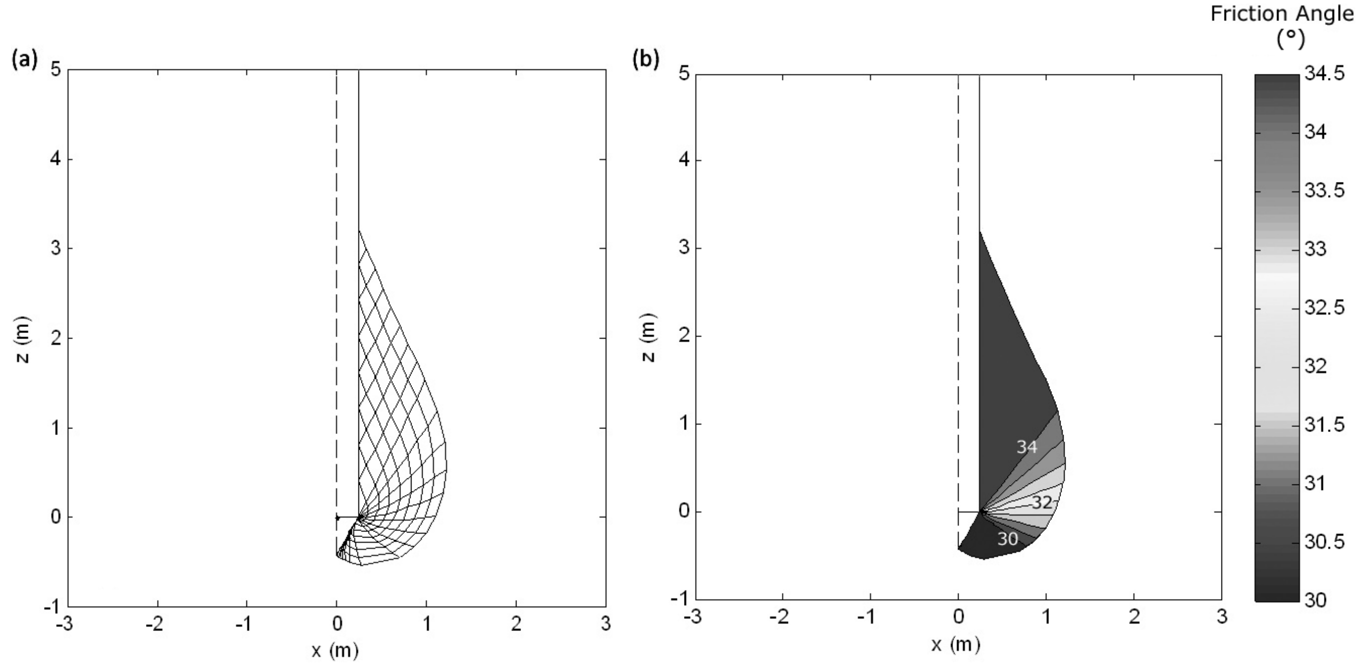
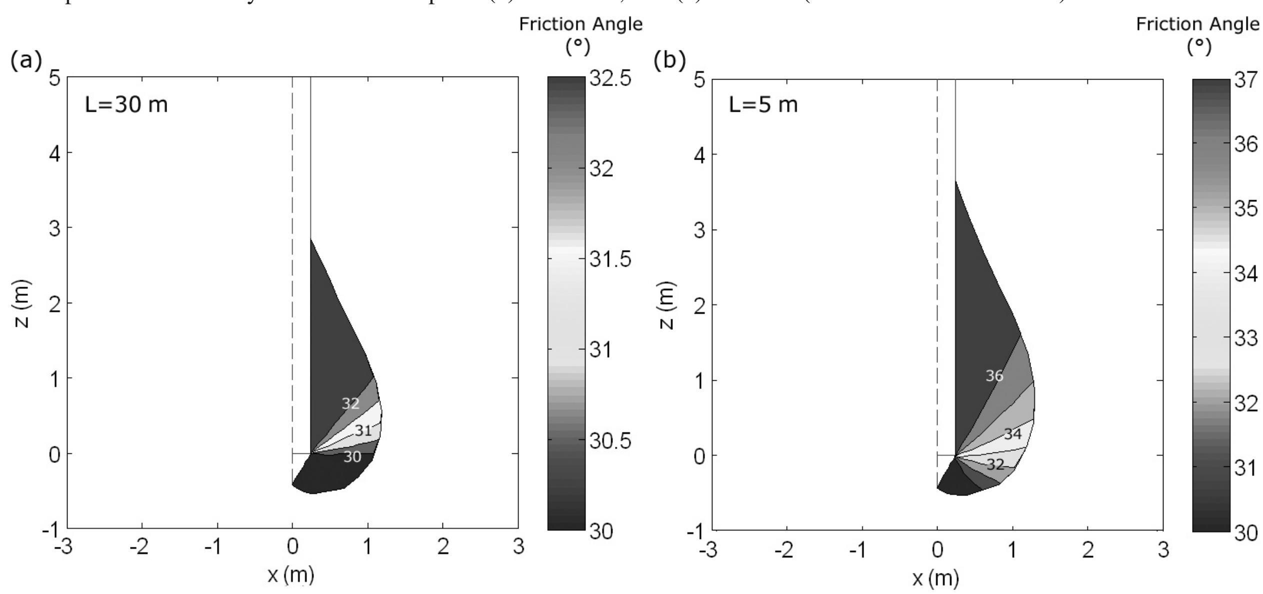


Fig. 6. Comparison of the analyzed cases of two piles: (*a*) L = 30 m, and (*b*) L = 5 m (D = 0.5 m for both cases).



the critical state and peak friction angles. Also, a slight curvature can be observed in the results of this work indicating a transition from the two extreme values corresponding to the peak and the critical state friction angles when the pile length increases.

Verification by case histories

The end bearing capacity of piles has been computed for 14 pile load test case histories at different sites throughout the world for which CPT soundings were performed in the vicinity of the piles' locations. The database comprised those cases with similar soil condition, pile type, and shape, that is, cases of driven piles in sand with circular cross sections and closed ends. Required parameters to compute the bearing capacity of piles were obtained using the CPT and CPTu records. These parameters include the soil peak friction angle (according to Robertson and Campanella 1983) and relative density (according to Bolton and Gui 1993). Variation of the soil friction angle with the stress level was considered by making use of Bolton's (1986) equation (eq. [2]). A summary of case studies compiled for this work is summarized in Table 2.

Computations were also made using other methods by Vesić (1975) and Janbu (1976) based on a constant friction angle (peak or critical state) distribution in the soil around

Fig. 7. Variation of the end-bearing capacity of piles with pile length for $\phi_{c.s.} = 30°$ and $D_r = 50\%$.

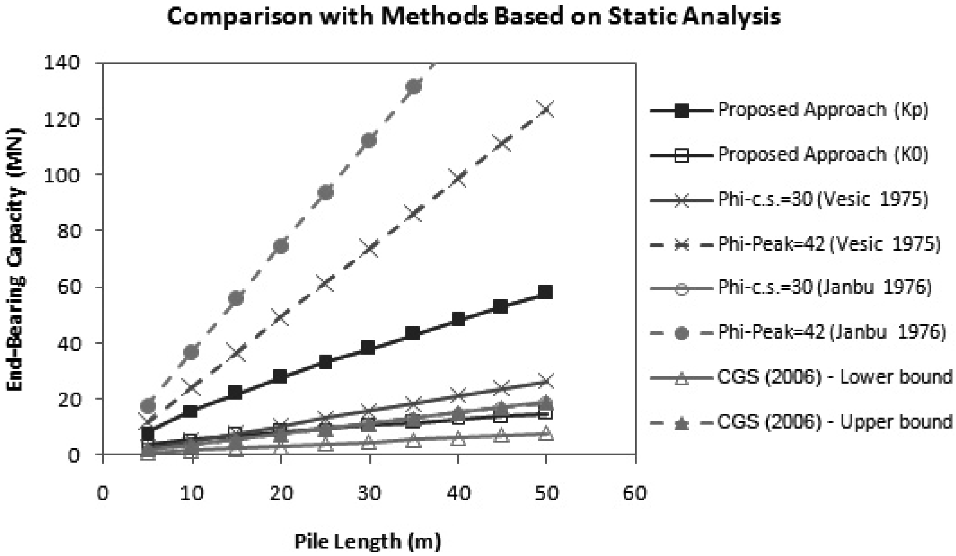


Table 2. Summary of pile cases records.

ID No.	Site	Reference	Soil type (at pile toe)	Diameter (mm)	Length (m)	Toe capacity (kN)	Avg. q_c (MPa)[a]	ϕ_p (°)	D_r (%)
1	Vancouver, B.C.	Campanella et al. (1989)	Sand	324	16.8	315	3	26	35.8
2	Vancouver, B.C.	Campanella et al. (1989)	Silt	324	31.1	180	2	18	33.8
3	San Francisco, Calif.	O'Neill (1988)	Sand	273	9.2	355	4	32	40.4
4	Baghdad, Iraq	Altaee et al. (1992)	Uniform sand	285	11	360	3	29	37.5
5	Los Angeles, Calif.	CH2M Hill (1987)	Dense sand	600	25.8	4050	17	35	44.3
6	Los Angeles, Calif.	Fellenius (1995)	Dense sand	600	32.6	3560	20	34	43.5
7	Taiwan	Yen et al. (1989)	Sand	609	34.25	1650	5	24	35.3
8	São Paulo, Brazil	Decourt and Niyama (1994)	Silty sand	500	8.7	2000	8	37	49
9	Victoria, Australia	Haustorfer and Pleisiotis (1988)	Dense sand	450	13.8	1900	6.0	32	40.4
10	São Paulo, Brazil	Albiero et al. (1995)	Silty sand	350	9.4	240	2	27	36.4
11	São Paulo, Brazil	Albiero et al. (1995)	Silty sand	400	9.4	310	2	27	36.4
12	Potenza, Italy	Apendino (1981)	Dense sand	508	35.85	3000	15	32	40.1
13	Potenza, Italy	Apendino (1981)	Dense sand	508	43	2600	16	31	39.3
14	Houston, Tex.	O'Neill (1981)	Sandy clay	273	13	245	3	28	36.8

[a]Average of q_c in the "influence zone".

the toe. It should be noted that the critical state friction angle was calculated by inverting Bolton's (1986) equation, knowing the peak friction angle and the soil relative density.

The graphical representation of the mobilized friction angle distribution and the stress characteristics net for some cases are illustrated in Fig. 8. Figure 8*a* shows the results obtained for case 3, a driven pile in sand located in San Francisco, Calif. The measured and computed values of the toe-bearing capacity are 355 and 432 kN, respectively. Figure 8*b* represents the results for case 4, a driven pile in uniform sand, tested in Baghdad, Iraq. The measured ultimate toe capacity was 360 kN whereas the computed value is 322.9 kN. Figure 8*c* shows the results obtained for case 7, a driven pile tested in Taiwan. In this case, the ultimate toe capacity was measured to be 1650 kN and the computed value is 1990 kN, which is slightly higher than the actual value. Figure 8*d* shows the results obtained for case 11, a driven pile in silty sand, tested in São Paolo, Brazil. The measured ultimate toe capacity is 310 kN whereas the computed value is 463 kN. It should be noted that in all these figures, only the plastic region around the pile toe is shown, which is roughly 5–10 times the pile diameter.

Predictions made using other methods have been compared with the proposed approach, based on the stress characteristics method incorporating the stress level dependent soil friction angle. The range of measured to estimated end-bearing capacity ratio can be theoretically between zero and infinity; whereas, in an ideal case, it should be one. To validate the proposed approach, further comparisons with other methods based on the static analysis and direct CPT were made. Methods based on static analysis, which have been commonly referenced in geotechnical sources and addressed earlier, are those attributed to Vesić (1975), Janbu (1976), and Meyerhof (1976, 1983). Besides, for long-term analysis the pile toe-bearing capacity is governed by the effective stress method, and hence, the method of CGS (2006), was also included in comparisons. Also, re-

Fig. 8. Illustration of the stress characteristics net and variation of soil friction angle at failure in analyzed cases: (*a*) case 3; (*b*) case 4; (*c*) case 7; (*d*) case 11.

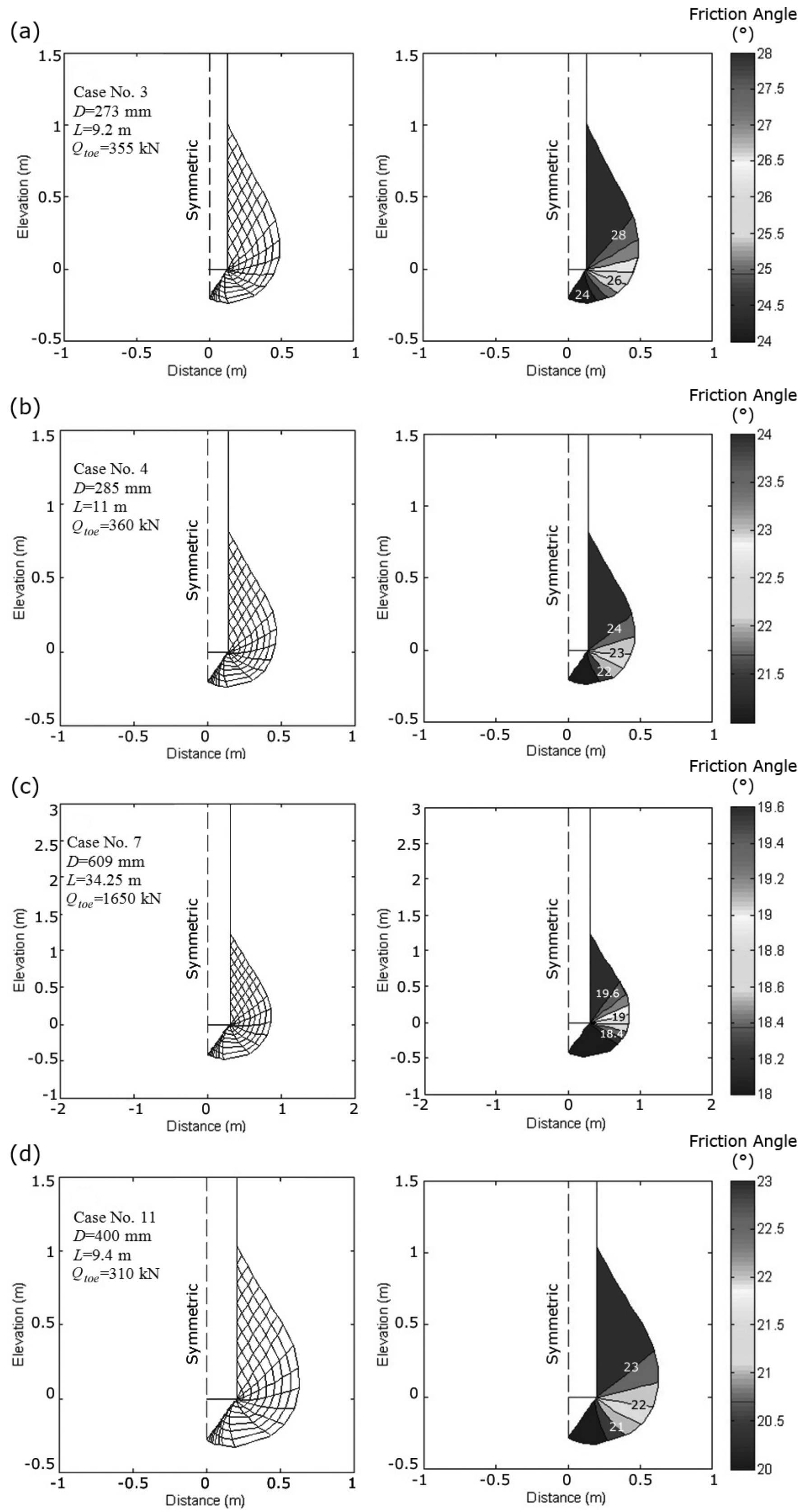
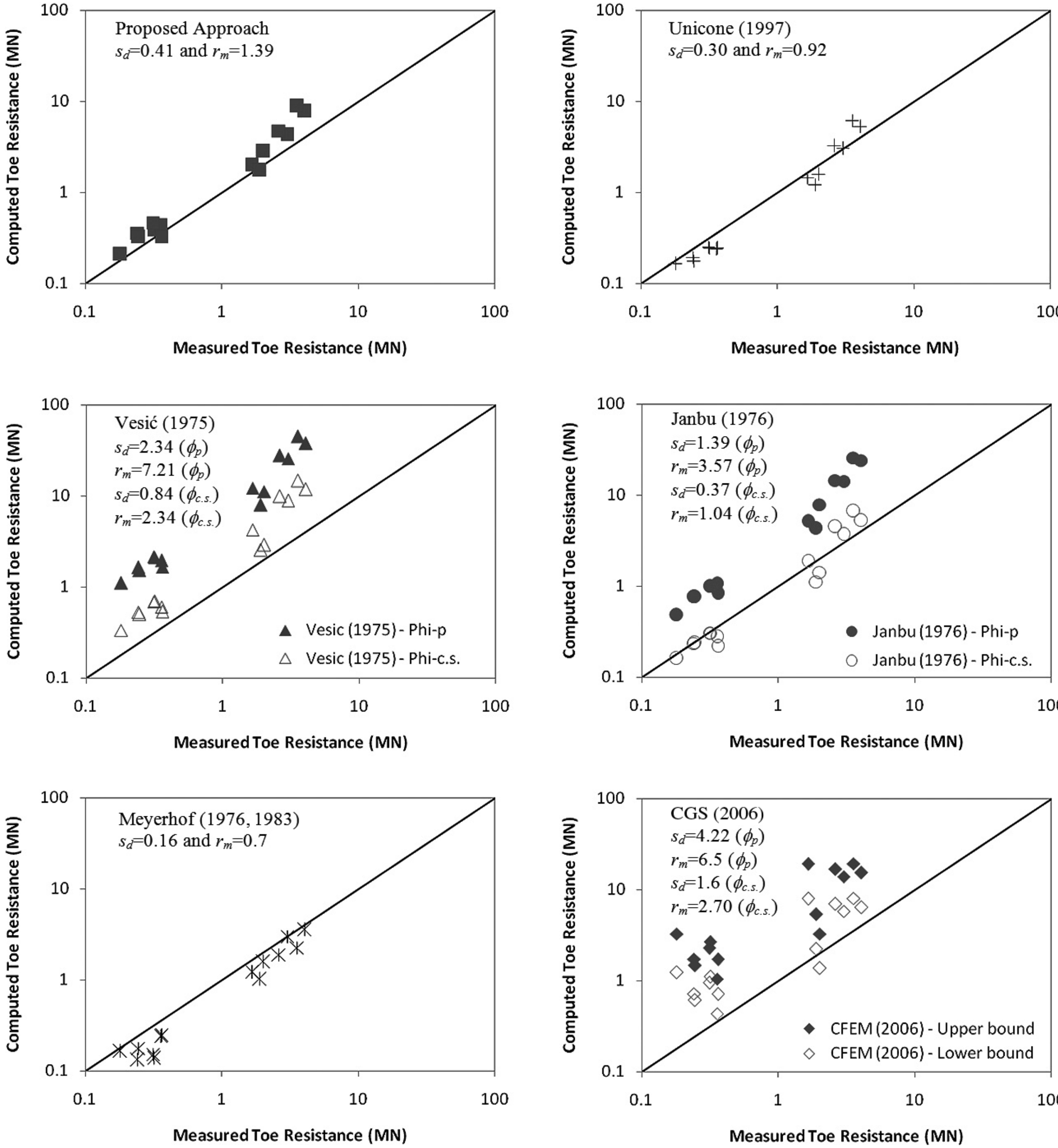


Fig. 9. Estimated versus measured end-bearing capacity by different methods.

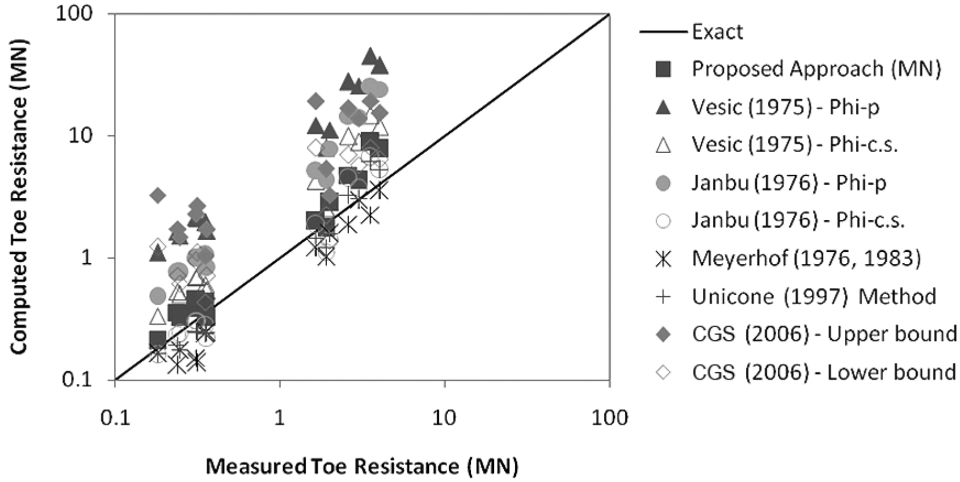


garding the compiled case studies with continuous records of the CPT and CPTu soundings, the recent CPTu method, known as the Unicone (1997) method (Eslami and Fellenius 1997; Fellenius 2006), has been employed for verification.

In statistics, the standard deviation and the mean value give the accuracy and precision of a prediction method. The normal distribution can be defined as a distribution with the density defined by the following equation:

$$[13] \qquad f(r) = \frac{1}{\sqrt{2\pi}s_{\mathrm{d}}}\exp\left[-\frac{1}{2}\left(\frac{r-r_{\mathrm{m}}}{s_{\mathrm{d}}}\right)^2\right]$$

Fig. 10. Computed versus measured end-bearing capacity of piles using different methods on one plot.



where $f(r)$ is the normal distribution density, r is the ratio of the measured to the computed end-bearing capacities, s_d is the standard deviation of these values, and r_m is the mean value.

To evaluate different methods utilized in estimation of the end-bearing capacity of piles, in each case, the standard deviation and the mean value of the measured to the predicted values have been computed and shown on each figure. Ideally, a mean value of one and a standard deviation of zero show the exact prediction. However, the method is better when the standard deviation is closer to zero and the mean value to unity. Statistical analyses revealed that the proposed approach can provide relatively good estimates of the end-bearing capacity of piles in comparison to other methods. Figure 9, shows a comparison of the results obtained using different methods. A combined plot of all computed values based on different methods is also presented in Fig. 10.

An insight into the results obtained using different methods shows that methods based on static analysis and the effective stress approach (i.e., Vesić 1975; Janbu 1976) assuming the peak friction angle and the method of CGS (2006) give overestimated predictions. In contrast, direct use of the CPT method, suggested by Meyerhof (1976), gives rather underestimated values. The Unicone (1997) method and Janbu (1976) method, based on the critical state friction angle, as well as the proposed approach, based on the stress-level-dependent method of stress characteristics, show the best agreement with measured values.

Figure 11 shows the variation of the bearing capacity ratio, defined as the ratio of the computed to measured values, for the aforementioned methods. It can be observed that, in general, the results obtained using the proposed approach are still overestimated in most cases, although they can capture the actual end bearing with relatively good accuracy.

The source of this overestimation can be related to the nonassociative nature of the soil. To achieve better results, the effect of nonassociativity is considered by making use of the equivalent or apparent friction angle, ϕ^*, originally defined by Vermeer (1990) and Drescher and Detournay (1993), while Bolton's (1986) equation can be used for the dilation angle, ν. These two equations can be combined to find an equivalent friction angle in which the effect of stress level as well as the soil nonassociativity are involved at the same time.

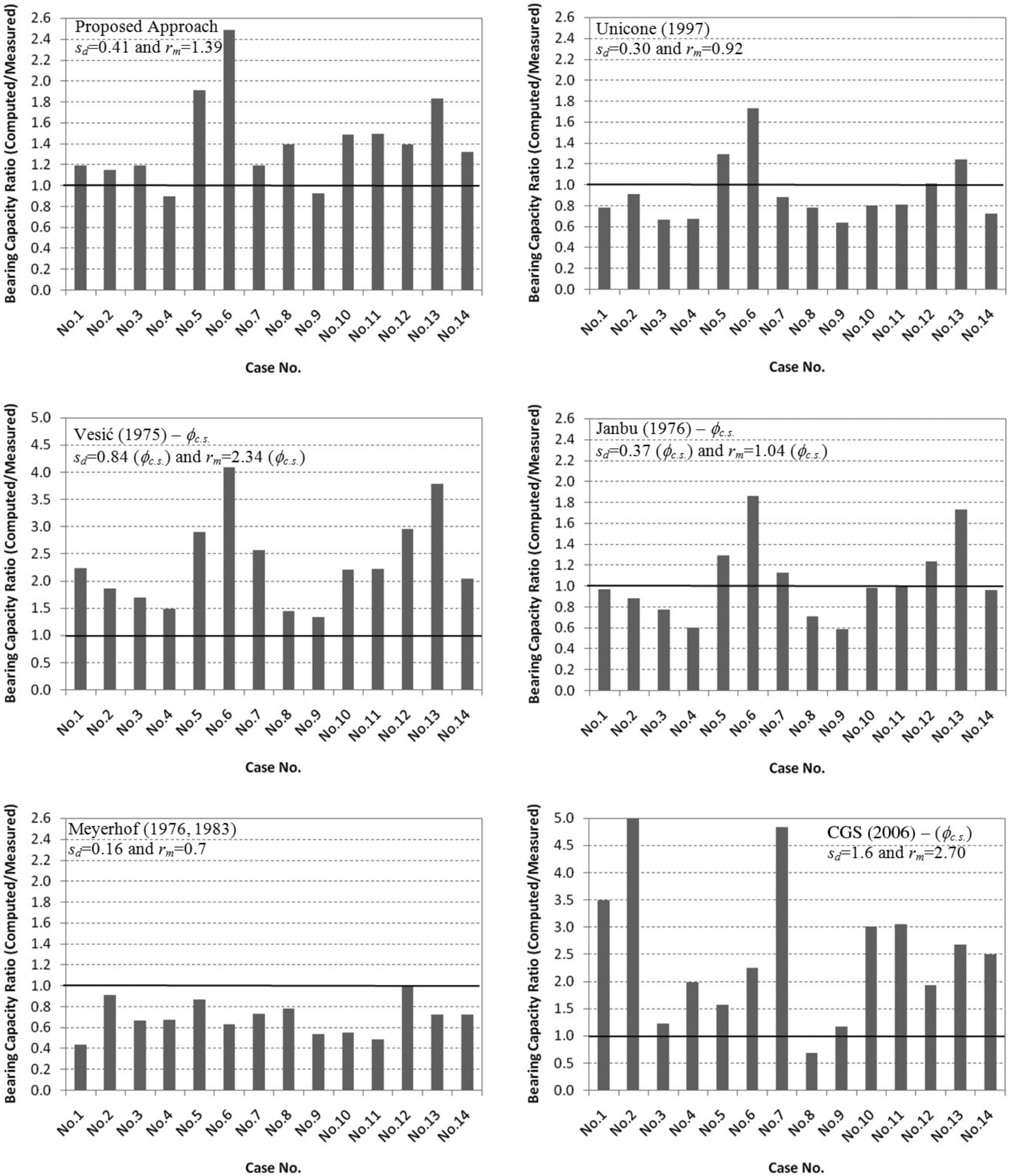
It is important to note that the definition of the equivalent friction angle, ϕ^*, was made only to define the plastic potential surface, and the corresponding yield surface remains unchanged. There is no indication of the plastic flow in the method of stress characteristics; only the parameters of the selected yield criterion (Mohr–Coulomb) are required. However, it is apparent that at the ultimate load, some plastic strains are required to mobilize the resisting friction angle in the soil mass and, hence, flow rule is certainly important. In the method of stress characteristics, it is assumed that plastic regions are formed at failure, in some regions adjacent to the base of the foundation. These regions must coincide with plastically deforming body and therefore, it seems that an "equivalent associated flow rule" is not an invalid assumption. Therefore, considering the highly sheared regions around the pile toe, an equivalent associated flow rule can be assumed to govern the material behavior and, hence, an equivalent yield criterion with herein-defined equivalent friction angle, ϕ^*. Then, the method of stress characteristics can be applied with substitution of the friction angle, ϕ, with the equivalent friction angle, ϕ^*.

Figure 12 illustrates a comparison between the results obtained by associated and nonassociated flow rule assumptions. The data shows an average of 1.17 (with seven cases showing pile end capacity ratios of 1.0–1.2) and a standard deviation of 0.27, indicating a rather good prediction. It is evident that the nonassociativity assumption gives a better estimation of the end-bearing capacity. Therefore, this additional assumption has been included in the proposed approach for the rest of this work.

Discussion

It was shown that, in static analysis methods, N_t is a function of the soil friction angle, which is a stress-level-dependent parameter and so should be N_t. On the other hand, better results can be achieved if the effect of the nonassociativity is considered in computation of the equivalent friction angle. As a practical approach, regarding the nonlinearity of the Mohr–Coulomb failure envelope and the highly nonassociative nature of the soil under relatively high stresses at the pile toe, the bearing capacity of deep foundations can be expressed by the following rather simple equation:

Fig. 11. Representation of the bearing capacity ratio in different methods.



[14] $q_{\text{ult}} = N_t^* q_z$

where N_t^* is the toe-bearing capacity factor with corrections for the stress level and the nonassociativity effects. This last equation is the same as the equation suggested by the CGS (2006) method and methods based on static analysis. Both effects of the weight and the overburden pressure are included in a single factor (i.e., N_t^*). There is only one major differ-

Fig. 12. Comparison of associative and nonassociative cases: (*a*) predicted and (or) measured values; (*b*) distribution of the results for both cases in comparison with the normal distribution curve for nonassociative case.

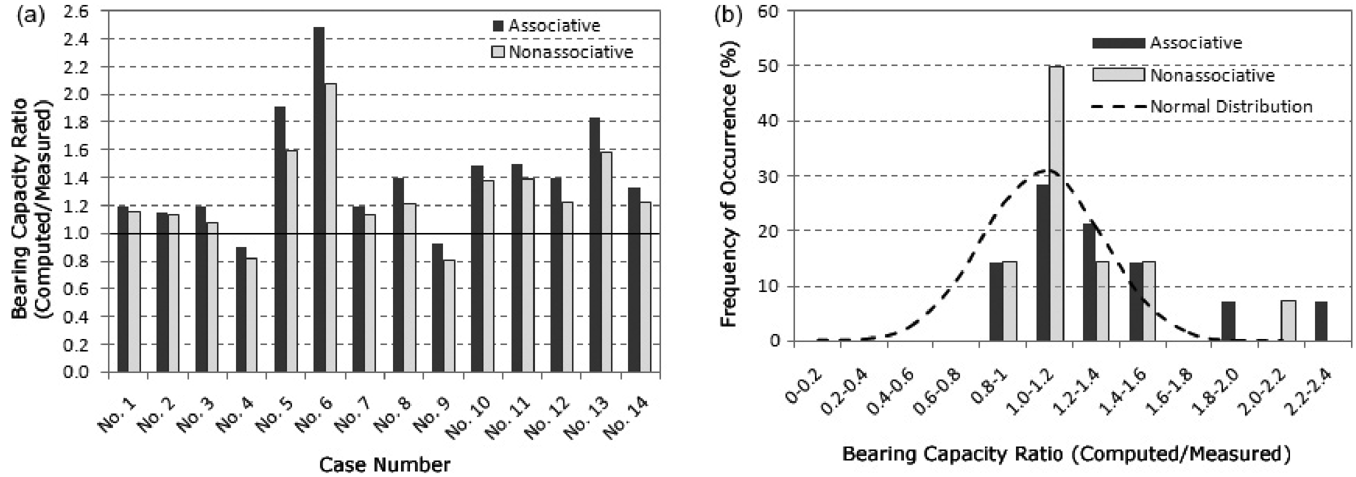


Fig. 13. Variation of the end-bearing capacity factor, N_t^*, with depth and soil friction angle in the proposed method.

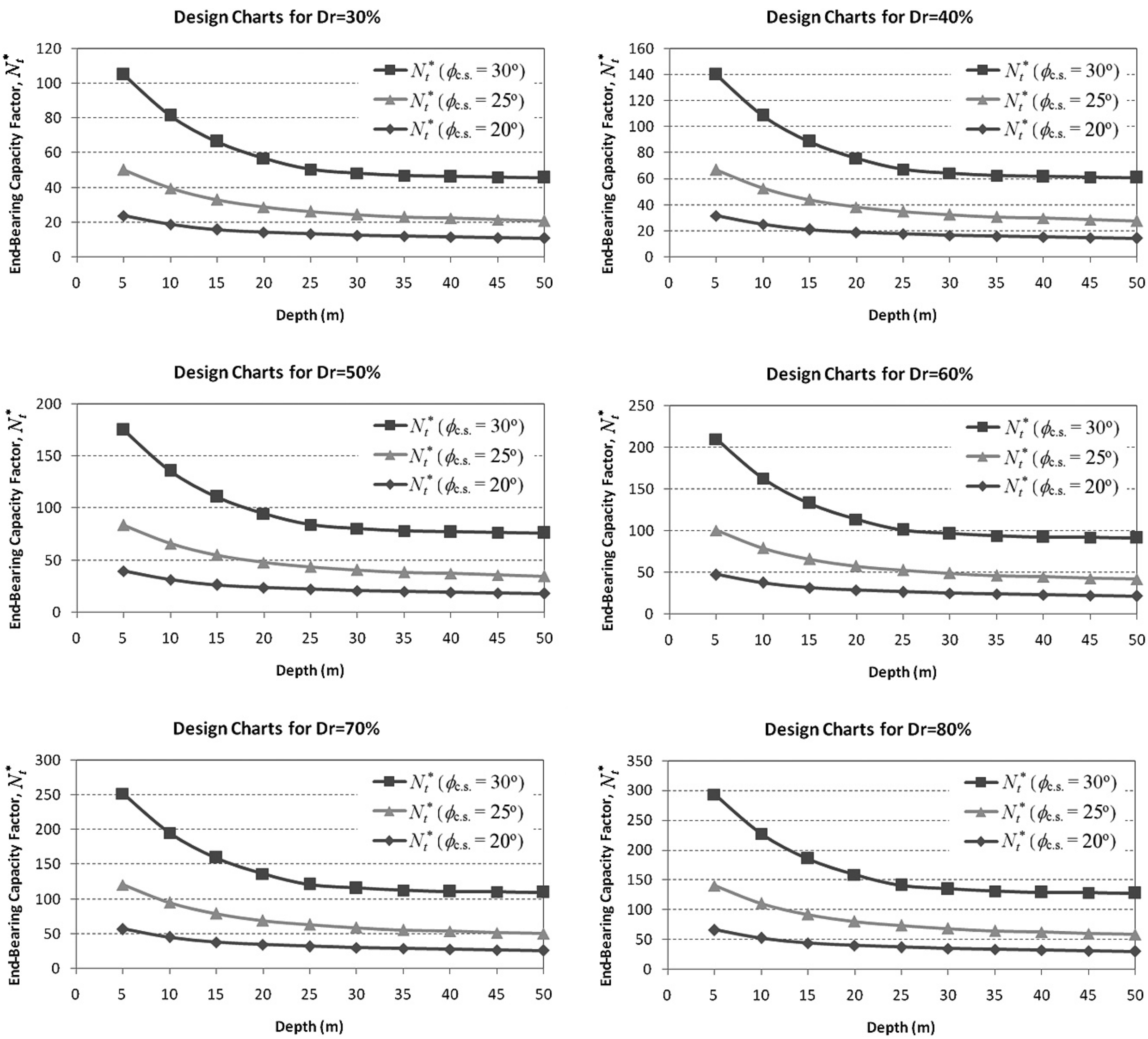


Table 3. Results of the analyzed cases for direct use of the design chart.

ID no.	Diameter (mm)	Length (m)	Toe capacity (kN)	Avg. q_c (MPa)	ϕ_p (°)	D_r (%)	$\phi_{c.s.}$ (°)	N_t^* (graph)	Computed to measured ratio
1	324	16.8	315	3	26	35.8	19	20	1.01
2	324	31.1	180	2	18	33.8	12	15	—
3	273	9.2	355	4	32	40.4	24	55	1.09
4	285	11	360	3	29	37.5	21	36	1.07
5	600	25.8	4050	17	35	44.3	26	51	1.42
6	600	32.6	3560	20	34	43.5	26	54	2.12
7	609	34.25	1650	5	24	35.3	18	17	1.16
8	500	8.7	2000	8	37	49	27	92	1.23
9	450	13.8	1900	6.0	32	40.4	24	50	0.95
10	350	9.4	240	2	27	36.4	20	32	1.39
11	400	9.4	310	2	27	36.4	20	32	1.43
12	508	35.85	3000	15	32	40.1	24	37	1.15
13	508	43	2600	16	31	39.3	24	33	1.39
14	273	13	245	3	28	36.8	21	34	1.25

ence that the soil friction angle is assumed to be constant in the conventional form of this equation. Such representation seems to be more applicable regarding the fact that the two contributors of the bearing capacity (i.e., the surcharge and the weight) terms cannot be easily separated in deep foundations. Moreover, there is no further need for the superposition, which could be unsafe in a nonlinear Mohr–Coulomb failure envelope. A number of analyses for pile foundations of different depths were carried out. Figure 13 shows the variation of the end-bearing capacity factor, N_t^*, with the embedment depth. The curves have been developed for soils of γ = 16 kN/m^3 to cover a wide range of sands at different relative densities ranging between 30% and 70%. Pile diameter was assumed to be 0.5 m, which is a typical value in practice. Variation of the soil friction and dilation angles was assumed to be governed by Bolton's (1986) equation and the effect of nonassociativity was considered by the apparent (or equivalent) friction angle in analyses. It can be well observed that the end-bearing capacity factor, N_t^*, is a function of the embedment depth and it decreases as the pile length increases. The graphs were developed for different critical state friction angles (i.e., $\phi_{c.s.}$ = 20°, 25°, and 30°). Values of the bearing capacity factor, N_t^*, were not extended to sands with higher critical state friction angles because such soils possess relatively high strengths and seldom require pile enhancement in practice. Lower values of the soil critical state friction angle correspond to loose soils in which there is no significant change in the maximum mobilized friction angle with the stress level. As a result, the curvature of the Mohr–Coulomb failure envelope is very slight and so is the variation of N_t^*. Further inspection of these graphs reveals that the change in the end-bearing capacity factor, N_t^*, is rapid for embedment depths between 10 and 20 m. Below a depth of around 30 m, on the other hand, the bearing capacity factor, N_t^*, tends to a relatively constant value with a slight variation over the depth.

To show the feasibility of the design charts, computations have been made for 13 case studies out of 14, based on appropriate assumptions using the developed design charts. Linear interpolation was done to compute the bearing capacity factor, N_t^*, where required. Table 3 shows a summary of the computed values. In eight cases (62% of 13 cases), the relative error is less than 25%. In four cases (31% of 13 cases), the relative error ranges between 25% and 50%. Only in one case does the relative error exceed 50%. Therefore, it can be observed that the graphs can be reasonably used in prediction of the end-bearing capacity of driven piles in sand.

Conclusions

An application of the standard bearing capacity equation for determining the bearing capacity of a pile base has been reinvestigated. The nonlinearity of the Mohr–Coulomb envelope results in dependency of the bearing capacity factor, N_γ, on the diameter of the pile base. It is noted that the validity of the superposition assumption cannot always be guaranteed. The study reveals that the bearing capacity can be estimated precisely if a more rational approach is used rather than using the principle of superposition. The method of stress characteristics, by incorporating the dependency of friction angle on the stress level, has been employed for determining the ultimate bearing capacity of the pile base. The dependency of the friction angle on the stress level has been included by using the empirical relationship proposed by Bolton. The comparison of the results obtained from the analysis with the various methods commonly used in practice, namely, static analysis, effective stress approach, and direct CPT methods, demonstrates clearly the usefulness of the proposed theoretical approach. It is noted that the results from the proposed approach lie generally within the common range of results suggested by the different methods. The bearing capacity of the pile base has been found to vary in a nonlinear fashion with depth.

Fourteen case studies of driven piles (*i*) comprising different depths and diameters, and (*ii*) embedded in different types of sand were examined. The present approach as well as different conventional methods, namely, static analysis, the effective stress approach, and direct usage of in situ test results, were applied to compute the bearing capacity of the pile base. The comparisons reveal that the stress-level-dependent method of stress characteristics can provide quite accurate predictions. The theoretical results generally overestimate the bearing capacity. The nonassociative nature of the soil around the pile toe was supposed to be the source of this overestimation. An equivalent value of the mobilized soil friction angle, for a given combination of peak friction angle and dilatancy angle, has been used to apply the nonassociative flow rule. Predictions made using this approximation showed better consistency with measurements.

Eventually, the bearing capacity of the pile base in sand was expressed in terms of a rather simple equation involving the single bearing capacity factor, N_t^*. Nondimensional design charts have been developed for different combinations of the values of (*i*) critical state friction angles, (*ii*) relative density, and (*iii*) embedment depths that are commonly confronted in practice.

The research has revealed that the bearing capacity of the pile base in sand can be reasonably predicted by the proposed approach, which incorporates (*i*) the dependency of friction angle on the stress level, and (*ii*) the nonassociativity of soil.

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